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Minimal Dynamical Symmetry Breaking of the Electroweak Interactions and m_{top}

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Abstract

We review the recent idea of a mechanism for dynamically breaking the symmetries of the electroweak interactions which relies upon the formation of condensates involving the conventional quarks and leptons, such as the top quark. In particular, such a scheme would indicate that the top quark is heavy, greater than or of order 200 GeV, and gives further predictions for the Higgs boson mass. It may be extended to a fourth generation with new strong TEV scale flavor–interactions.

We now know from CDF that $m_{top} \gtrsim 89$ GeV. The top quark is thus the most strongly coupled fermion to the *agent or dynamics which breaks the electroweak interactions* and the CDF lower limit implies a Higgs–Yukawa coupling constant, $g_{top} \gtrsim 0.5$. A large m_{top}, moreover, leads to difficulties for conventional extended technicolor, and even walking technicolor for very large m_{top} ultimately requires fine–tuning. This, in turn, suggests that the top quark might, itself, play a fundamental role in the breaking of electroweak symmetries [1 – 3] by acting as a "techniquark," as a consequence of some new interaction.

Nambu first proposed that the symmetry breaking of the electroweak interactions arises in analogy to chiral symmetry breaking by a pairing of top quarks [1]. Bardeen, Hill and Lindner, (BHL), inspired by the work of Nambu, subsequently gave a technically complete implementation of the top–condensate idea [3] and obtained the first realistic predictions in a minimal scheme. BHL straightforwardly implemented a BCS or Nambu–Jona-Lasinio (NJL) mechanism in which a new fundamental interaction associated with a high energy scale, Λ, is used to trigger the formation of a low energy condensate, $\langle \bar{t}t \rangle$. In this scheme only the known dynamics of the standard model is incorporated. BHL are ultimately able to derive precise predictions for

m_{top} and m_{Higgs} in this scheme by considering the role of the renormalization group. The usual single Higgs–doublet standard model emerges as the low energy effective Lagrangian, but with new constraints that lead to the nontrivial predictions.

We will summarize here the BHL analysis. In particular the application of the renormalization group (RG) will be considered only briefly after a summary of more familiar the Schwinger–Dyson (SD) analysis of the model. It should be noted that the RG analysis is more general: by keeping only the terms in the RG that pertain to the fermion loops one reproduces exactly the SD results, yet one can include additional effects in the RG easily to go beyond the limited large–N_c SD analysis.

Consider, for discussion, the approximation in which all quarks and leptons other than the top quark are massless (a simple generalization of eq.(1) accomodates all nonzero fermion masses and mixing angles, but leads to no new predictions). We may then define the theory at the scale Λ to be:

$$L = L_{kinetic} + G(\bar{\Psi}_L^{ia} t_{Ra})(\bar{t}_R^b \Psi_{Lib}) \tag{1}$$

Here $\Psi_L = (t, b)_L$ and i runs over $SU(2)_L$ indices, (a, b) run over color indices. $L_{kinetic}$ contains the usual gauge invariant fermion and gauge boson kinetic terms. We first consider a solution to the model based upon the effects of the fermionic determinant alone, *i.e.*, a fermion bubble approximation. This is equivalent to a large–N_{color} expansion in the limit in which the QCD coupling constant is set to zero, and it captures nonperturbative features of the theory from the point of view of a small–coupling constant expansion.

We thus demand a solution to the gap equation for the induced top quark mass:

$$m_t = -\frac{1}{2} G \langle \bar{t}t \rangle = 2GN_c m_t \frac{i}{(2\pi)^4} \int d^4l \, (l^2 - m_t^2)^{-1} \tag{2}$$

or:

$$1 = \frac{GN_c}{8\pi^2} \left(\Lambda^2 - m_t^2 \ln(\Lambda^2/m_t^2) \right) . \tag{3}$$

which has solutions for sufficiently strong coupling, $G \geq G_c = 8\pi^2/N_c\Lambda^2$ where G_c is the "critical" coupling constant. We regard G and Λ as fundamental parameters of the theory and we solve for m_t. Normally, for very large Λ, perhaps of order the GUT scale 10^{15} GeV, we would expect the solution of this equation to produce a large mass, $m_t \sim \Lambda$ in the broken symmetry phase. We see that a solution for $m_t \sim M_W << \Lambda$ constitutes a fine–tuning problem in that $G^{-1} - G_c^{-1}$ must then be very small. This is, indeed, the usual fine–tuning or gauge hierarchy problem of the standard model. The gap equation contains a quadratic divergence, corresponding to the usual Higgs mass quadratic divergence in the standard model. *However, the fine–tuning problem will be isolated in the gap equation, i.e.*, once we tune G to admit the desirable solution we need cancel no other quadratic divergences in other amplitudes to any given order in perturbation theory.

If we now consider the sum of leading large-N_c scalar channel fermion bubbles generated by the interaction eq.(1) we find:

$$\Gamma_s(p^2) = \frac{1}{2N_c}\left[(p^2 - 4m_t^2)(4\pi)^{-2}\int_0^1 dx \log\left\{\Lambda^2/(m_t^2 - x(1-x)p^2)\right\}\right]^{-1} \quad (4)$$

Γ_s is the propagator for a dynamically generated 0^+ boundstate, a scalar composite particle composed of $\bar{t}t$. In particular, owing to the pole at $p^2 = 4m_t^2$, we see that the theory predicts the boundstate mass of $2m_t$. This is a standard result for the Nambu–Jona-Lasinio model. We emphasize that this boundstate is the physical, observable low energy Higgs boson. The prediction holds here only to leading order in $1/N_c$ in the absence of gauge boson corrections. We can also infer from eq.(4) that this particle is described by a field with a wave–function renormalization constant, Z_H, given by:

$$Z_H = \frac{N_c}{8\pi^2}\int_0^1 dx \log\left\{\Lambda^2/(m_t^2 - x(1-x)p^2)\right\} \quad (5)$$

This is *a relativistic boundstate*, and normal intuition from nonrelativistic potential models does not apply. In fact, the compositeness of this state is reflected by the behavior of Z_H:

$$Z_H \to 0 \qquad \text{as} \qquad -p^2 = \mu^2 \to \Lambda^2. \quad (6)$$

The essential point is embodied in eq.(6) and this allows us to give a more precise determination of the top mass upon considering the full renormalization group behavior of the complete theory.

Since this mechanism is indeed a dynamical breaking of the continuous $SU(2) \times U(1)$ symmetry it implies the existence of Goldstone modes. Moreover, the symmetry breaking transforms as $I = \frac{1}{2}$ and will produce the same spectrum of Goldstone bosons as in the standard model Higgs–sector. Of course, we have a dynamical Higgs–mechanism and the gauge bosons acquire masses by "eating" the dynamically generated Goldstone poles. We obtain a second prediction of the theory in the form of a relation between the W boson mass and the top quark mass as follows.

Consider now the inverse propagator of the gauge bosons. We rescale fields to bring the gauge coupling constants into the gauge boson kinetic terms, *i.e.*, we write the kinetic terms in the form $(-1/4g^2)(F_{\mu\nu})^2$. It is useful to write the induced inverse W boson propagator in the form:

$$\frac{1}{g_2^2}D^W_{\mu\nu}(p)^{-1} = (p_\mu p_\nu/p^2 - g_{\mu\nu})\left[\frac{1}{\bar{g}_2^2(p^2)}p^2 - \bar{f}^2(p^2)\right]. \tag{7}$$

The W boson mass is the solution to the the mass–shell condition:

$$M_W^2 = p^2 = \bar{g}_2^2(p^2)\bar{f}^2(p^2) \tag{8}$$

while the Fermi constant is the zero–momentum expression:

$$\frac{G_F}{\sqrt{2}} = \frac{1}{8\bar{f}^2(0)} \tag{9}$$

In the bubble approximation we find:

$$\begin{aligned}\frac{1}{\bar{g}_2^2(p^2)} &= \frac{1}{g_2^2} + N_c(4\pi)^{-2}\int_0^1 dx\ 2x(1-x) \\ &\quad \times \log\left\{\Lambda^2/(xm_b^2 + (1-x)m_t^2 - x(1-x)p^2)\right\}\end{aligned} \tag{10}$$

and:

$$\begin{aligned}\bar{f}^2(p^2) &= N_c(4\pi)^{-2}\int_0^1 dx\ (xm_b^2 + (1-x)m_t^2) \\ &\quad \times \log\left\{\Lambda^2/(xm_b^2 + (1-x)m_t^2 - x(1-x)p^2)\right\}\end{aligned} \tag{11}$$

A quantitative prediction for m_t in terms of G_F results when eq.(9) is combined with eq.(11):

$$\begin{aligned}\bar{f}^2(0) = \frac{1}{4\sqrt{2}G_F} &\approx N_c(4\pi)^{-2}\int_0^1 (1-x)m_t^2 \log\left\{\Lambda^2/((1-x)m_t^2)\right\} \\ &\approx \frac{1}{2}N_c(4\pi)^{-2}m_t^2\log\{\Lambda^2/m_t^2\}\end{aligned} \tag{12}$$

For example, with $\Lambda = 10^{15}$ GeV one finds $m_t \approx 165$ GeV (see Table I).

To what extent is this an accurate prediction for m_t? For one, it is valid only in leading order of $1/N_c$ with $g_3 = 0$. This result, moreover, neglects the full dynamical effects of gauge bosons and the composite Higgs boson, which should be included in the renormalization group running below the scale Λ. We note that this result is substantially less than the full RG–improved standard model result as described below. Analogous results are obtained for the neutral gauge boson masses, but they contain no additional information beyond that described here, a consequence of the conventional $I = \frac{1}{2}$ breaking mode. Moreover, the usual ρ parameter relationship for m_t emerges.

The dynamically generated scalar boundstates are point–like fields on all scales $\mu << \Lambda$ and are described by the following effective Lagrangian:

$$\begin{aligned} L &= L_{kinetic} + (\bar{\Psi}_L t_R H + h.c.) \\ &\quad + Z_H |D_\mu H|^2 - m_H^2 H^\dagger H - \frac{\lambda_0}{2}(H^\dagger H)^2 \end{aligned} \tag{13}$$

We include here the gauge invariant kinetic terms of the Higgs doublet and its induced quartic interaction coming from top quark loops, as well as the wave–function normalization constant, Z_H.

In the present case, however, the Higgs field is dynamical with a vanishing wave–function renormalization constant at the scale $\mu \sim \Lambda$. That is, we have the following conditions at Λ (in terms of the unconventional normalization):

$$Z_H \propto N_c \ln(\Lambda/\mu) \rightarrow 0|_{\mu\rightarrow\Lambda}. \tag{14}$$

$$\lambda_0 \propto N_c \ln(\Lambda/\mu) \rightarrow 0|_{\mu\rightarrow\Lambda} \tag{15}$$

Note that as $\mu \rightarrow \Lambda$ the Lagrangian of eq.(13) involves only an auxiliary (non–propagating) field H with a quadratic mass–term and linear coupling. Thus, integrating this field out reproduces eq.(1), which shows how the RG boundary conditions of eq.(14) and eq.(15) are consistency conditions of the theory. Consistency is only possible when $G > 0$, corresponding to an attractive interaction in eq.(1).

Conventionally one normalizes the kinetic terms of a field theory at any scale, μ, with a condition that the kinetic terms have free–field theory normalization. That is, we may exercise our freedom of rescaling the various fields, H, Ψ_L, t_R, *etc.*, to define the coefficient of $|D_\mu H|^2$ to be unity. In the present case $H \rightarrow H/\sqrt{Z_H}$. The physical coupling constants, such as top quark Higgs–Yukawa coupling, $\bar{g}_t$, and the quartic Higgs coupling, $\bar{\lambda}$, are then:

$$\bar{g}_t = \frac{1}{\sqrt{Z_H}}; \qquad \bar{\lambda} = \frac{1}{Z_H^2}\lambda_0 \tag{16}$$

It is clear from eqs.(16) that as $\mu \rightarrow \Lambda$ then $\bar{g}_t$ and $\bar{\lambda}$ diverge, while $\bar{g}_t^2/\bar{\lambda} \rightarrow$ *constant*.

To obtain an RG improvement over the large–N_c NJL model we may utilize these boundary conditions on $\bar{g}_t$ and $\bar{\lambda}$ and the full β-functions (neglecting light quark masses and mixings) of the standard model. To one–loop order we have the full RG equations:

$$16\pi^2 \, \frac{d\bar{g}_t}{dt} = (\frac{9}{2}\bar{g}_t{}^2 - 8\bar{g}_3{}^2 - \frac{9}{4}\bar{g}_2{}^2 - \frac{17}{12}\bar{g}_1{}^2)\bar{g}_t \tag{17}$$

and, for the gauge couplings:

$$16\pi^2 \, \frac{d\bar{g}_i}{dt} = -c_i \, \bar{g}_i{}^3 \tag{18}$$

with

$$c_1 = -\frac{1}{6} - \frac{20}{9}N_g; \quad c_2 = \frac{43}{6} - \frac{4}{3}N_g; \quad c_3 = 11 - \frac{4}{3}N_g \tag{19}$$

where N_g is the number of generations and $t = \ln \mu$.

The precise value of the top quark mass is determined by running $\bar{g}_t(\mu)$ from very high values at a given compositeness scale Λ down to the mass–shell condition $\bar{g}_t(m_t)v/\sqrt{2} = m_t$. The nonlinearity of eq.(17) focuses a wide range of initial values into a small range of final low energy results. For an estimate one can assume that the gauge couplings are constant, which indicates why the solutions are attracted toward the effective low energy fixed–point [4]:

$$\bar{g}_t{}^2(\mu) \approx \frac{16}{9} \, \bar{g}_3{}^2(\mu) \tag{20}$$

The action of the infrared fixed–point makes the top quark mass prediction very insensitive to the initial high values of the coupling constant close to Λ. The uncertainties of higher orders can be viewed as an uncertainty in the precise position of Λ, and the fixed point behavior implies that m_t is determined up to $O(\ln\ln\Lambda/m_t)$ sensitivity to Λ. In Table I we give the resulting physical m_{top} obtained by a numerical solution of the renormalization group equations as a function of Λ.

The Higgs boson mass will likewise be determined by the evolution of $\bar{\lambda}$ given by:

$$16\pi^2 \frac{d\bar{\lambda}}{dt} = 12(\bar{\lambda}^2 + (\bar{g_t}^2 - A)\bar{\lambda} + B - \bar{g_t}^4) \tag{21}$$

where:

$$A = \frac{1}{4}\bar{g_1}^2 + \frac{3}{4}\bar{g_2}^2; \quad B = \frac{1}{16}\bar{g_1}^4 + \frac{1}{8}\bar{g_1}^2\bar{g_2}^2 + \frac{3}{16}\bar{g_2}^4 \tag{22}$$

The resulting prediction of the full standard model analysis is a top quark mass that might be considered large in comparison to upper limits derived from global analyses of all electroweak data that pertains to m_t. Typically these imply $m_t \lesssim 180$ to 200 GeV. If, ultimately, the physical top quark mass proves to be less than the theoretical prediction then it is still possible, albeit possibly less compelling, to maintain this mechanism by assuming that the gap equation is saturated by a fourth generation. The top quark then plays no important role itself in the symmetry breaking of the standard model and should have a mass between current lower bounds, but presumably much less than the predictions for the masses of the fourth generation.

Moreover, one might object to this scheme on the basis of naturalness and the fine–tuning that is implicit in demanding a solution in the limit $m_t << \Lambda$. We should therefore investigate whether there exist natural generalizations of the above mechanism and what kinds of natural theories might exist.

A supersymmetric extension of the model described above has been studied by Clark, Love and Bardeen [6]. One imagines an effective supersymmetric four–fermion interaction to exist on scales $\mu << \Lambda$ and supersymmetry is broken softly on a scale Δ. Here the quadratic divergence of the gap equation is essentially replaced by the SUSY soft–breaking scale Δ. Thus, if $\Delta \sim m_t$ and $G \sim 1/\Delta^2$ there is no large hierarchy. One generates a low energy effective Lagrangian which now contains the two Higgs bosons as demanded by supersymmetry and chirality. One of these (the one associated with top) is now composite with analogous compositeness conditions as above. The renormalization group improvement is thus similar to the preceding case the net result for $\Delta \sim 100$ GeV, $\Lambda \sim 10^{19}$ GeV is $m_t \approx 200$ GeV.

There is, however, a potential problem with schemes like this. In particular, solutions to the gap equation require $G \sim 1/\Delta^2$ while the four–fermion effective Lagrangian is viewed as valid up to scales $\mu \sim \Lambda$. This implies that G is extremely large on scales $\mu >> \Delta$ and thus there may be unitarity violations on scales large compared to Δ but small compared to Λ. While the fermion bubble sum implies that a partial unitarization has been performed in some channels, there could presumably be large violations in more complicated processes.

Perhaps the simplest solution to the naturalness problem is to consider theories in which $\Lambda \sim 1$ *to* 100 TeV. Then the top can probably no longer be upheld as the condensate since we see that m_t becomes $\sim$ 500 GeV and unacceptably large. However, a fourth generation is then workable. We emphasize that such has not been ruled out by neutrino counting at LEP and in fact, it is very reasonable in such a scheme to consider the see–saw mechanism to be operant at the electroweak scale. In this case a remarkable thing happens: light neutrinos go down to their experimental limits while heavy neutrinos go up to the electroweak scale [7]! Thus, we will consider a dynamical generation of the neutrino Majorana mass scale in the following. In fact, this is just a pure, ungauged BCS theory.

Consider a Lagrangian for right–handed neutrinos in isolation:

$$L = \bar{\nu}_R i\partial\!\!\!/ \nu_R + G(\bar{\nu}^c_{iR}\nu_{iR})(\bar{\nu}_{jR}\nu^c_{jR}) \tag{23}$$

where (^{c}) refers to charge conjugation, (i, j) are summed from 1 to N. In analogy to eq.(13) we introduce a composite field so that the effective Lagrangian in conventional notation takes the form:

$$L = \bar{\nu}_R i\partial\!\!\!/ \nu_R + |\partial\Phi|^2 - M^2\Phi^\dagger\Phi - \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 + (\kappa\bar{\nu}^c_{iR}\nu_{iR}\Phi + h.c.) \tag{24}$$

Now the RG equations are found to be [7]:

$$16\pi^2\frac{d}{dt}\kappa = 2N\kappa^3 + 4\kappa^3 \tag{25}$$

$$16\pi^2\frac{d}{dt}\lambda = 8N\kappa^2\lambda - 32N\kappa^4 + 8\lambda^2 \tag{26}$$

Note that, upon using the low energy effective Lagrangian and demanding that Φ develop a VEV v so that $\Phi = (v + \phi/\sqrt{2})\exp(i\chi/\sqrt{2}v)$, we see that χ is a massless Nambu–Goldstone mode and the residual Higgs–Majoron, ϕ, will have a mass $m^2_\phi = 2v^2\lambda$. The neutrinos will have Majorana masses of $m_M = 2\kappa v$

Consider the solution to eqs.(27, 28) in the large–N limit. We find:

$$\frac{1}{\kappa^2(\mu)} = \frac{2N}{(4\pi)^2}\log(\Lambda^2/\mu^2); \qquad \frac{1}{\lambda(\mu)} = \frac{N}{4(4\pi)^2}\log(\Lambda^2/\mu^2); \tag{27}$$

where we use the compositeness conditions, $\kappa(\mu \to \Lambda) \to \infty$, $\lambda(\mu \to \Lambda) \to \infty$. Hence, we obtain $m_\phi = 2m_M$, so the usual Nambu–Jona-Lasinio result holds in the Majorana or BCS case as well!

Incorporating this into a realistic theory involves more analysis. In general we will have additional quartic couplings of the dynamically generated Higgs boson and a term of the form $|H^2||\Phi|^2$. The full RG equations are now complicated and one must treat them numerically. This analysis has been performed by Hill, Paschos and Luty [7].

Such a theory is a novelty in terms of its dynamics, being a "Strong Broken Horizontal Gauge Symmetry." We have experience with the weak broken symmetries of the standard model and the strong confining gauge force of QCD, but it is unusual (albeit perfectly reasonable) to ponder a force that is, itself, broken yet sufficiently strong to drive the formation of chiral condensates. Thus a fourth generation with $\Lambda \sim 1$ TeV, is an intriguing possibility and we expect $m_{quarks} \sim 500$ GeV. We further note that nonminimal schemes can lead to multiple Higgs doublets in low energy effective theory, as analyzed by M. Suzuki and M. Luty [8].

In conclusion, As our discussion has indicated, the compositeness of the auxiliary Higgs field leads to predictions for the top quark and Higgs masses which are equivalent to effective fixed-point arguments. We have in this mechanism a *raison d'etre* for the single–Higgs doublet standard model with a heavy top quark.

Λ (GeV)	10^{19}	10^{15}	10^{11}	10^{7}	10^{5}
m_t (GeV); Fermion Bubble	144	165	200	277	380
m_t (GeV); Full RG	218	229	248	293	360
m_H (GeV); Full RG	239	256	285	354	455

Table I. Predicted m_{top} in two levels of increasingly better approximation as described in the text. "Fermion Bubble" refers only to the inclusion of fermion loops, equivalent to the conventional Nambu–Jona-Lasinio analysis, in which case $m_H = 2m_t$. All effects, including internal Higgs lines and electroweak corrections, are incorporated in the "Full RG" lines, and we include the m_H results. *Notice that the full renormalization effects cause* $m_H \neq 2m_t$.

References

1. Y. Nambu, "BCS Mechanism, Quasi-Supersymmetry, and Fermion Mass Matrix," Talk presented at the Kasimirz Conference, EFI 88-39 (July 1988); "Quasi-Supersymmetry, Bootstrap Symmetry Breaking, and Fermion Masses," EFI 88-62 (August 1988) in "*1988 International Workshop on New Trends in Strong Coupling Gauge Theories*," Nagoya, Japan, ed. Bando, Muta and Yamawaki.

2. W. J. Marciano, *Phys. Rev. Lett.* **62**, 2793 (1989);
V. A. Miransky, M. Tanabashi, K. Yamawaki, *Mod. Phys. Lett.* **A4**, 1043 (1989); *Phys. Lett.* **221B** 177 (1989).

3. W. A. Bardeen, C. T. Hill, M. Lindner, *Phys. Rev.* **D41**, 1647 (1990).

4. C. T. Hill, *Phys. Rev.* **D24**, 691 (1981);
C. T. Hill, C. N. Leung, S. Rao, *Nucl. Phys.* **B262**, 517 (1985).

5. T. Clark, S. Love, W. A. Bardeen, *Phys. Lett.* **B237**, 235 (1990).

6. C. T. Hill, E. A. Paschos, *Phys. Lett.* **B241**, 96 (1990);
C. T. Hill, M. Luty, E. A. Paschos, in preparation.

7. M. Suzuki, *Phys. Rev.* **D41**, 3457 (1990);
M. Luty, *Phys. Rev.* **D41**, 2893 (1990).